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## New logistical issues in using electric vehicle fleets with battery exchange infrastructure

Pitu Mirchandani<sup>a</sup>, Jonathan Adler<sup>a</sup>, Oli B.G. Madsen<sup>b</sup>

<sup>a</sup>*School of Computing, Informatics and Decision Systems Engineering  
Arizona State University, Tempe, Arizona 85281 United States*

<sup>b</sup>*Department of Transport, Technical University of Denmark, 2800, Kgs Lyngby, Denmark*

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### Abstract

There is much reason to believe that fleets of service vehicles of many organizations will transform their vehicles that utilize alternative fuels that are more sustainable. The electric vehicle (EV) is a good candidate for this transformation, especially which “refuels” by exchanging its spent batteries with charged ones. This paper discusses some new logistical issues that must be addressed by such EV fleets, principally the issues related to the limited driving range of each EV’s set of charged batteries and the possible detouring for battery exchanges. In particular, the paper addresses (1) the routing and scheduling of the fleet, (2) the locations of battery-exchange stations, and (3) the sizing of each facility. An overview of the literature on the topic is provided and some initial results are presented.

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Keywords; Electric vehicles; constrained shortest routes, vehicle routing; facility location, facility sizing

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### 1. Introduction

The environmental, geopolitical, and financial implications of the world’s dependence on gasoline-powered vehicles are well known and documented, and much has been done to lessen our dependence on gasoline. One thrust on this issue has been the embracing of the electric vehicles (EV) as an alternative to gasoline powered automobiles. These vehicles have an electric motor rather than a gasoline engine, and a battery to store the energy required for to move the vehicle. Governments and automotive companies have recognized the value of these vehicles in helping the environment [Hacker, Harthan, & Matthes 2009], and are encouraging the ownership of EVs through economic incentives. For many electric vehicles, such as the Nissan LEAF or Chevrolet VOLT, the current method of recharging the vehicle battery is to plug the battery into the power grid at places like the home or office [Bakker 2011]. Because the battery requires an extended period of time to recharge, this method has the implicit assumption that vehicle will be used only for driving short distances. EV companies are trying to overcome this limited range requirement with fast charging stations; locations where a vehicle can be charged in only a few minutes to near full capacity. Besides being much more costly to operate rapid recharge stations, the vehicles still take a more time to recharge than a standard gasoline vehicle would take to refuel [Botsford & Szczepanek, 2009]. These inherent problems, combined with a lack of refueling infrastructure, are inhibiting a wide-scale adoption of electric vehicles. These problems are especially apparent in longer trips, or inter-city trips. *Range anxiety*, when the driver is concerned that the vehicle will run out of charge before reaching the destination, is a major hindrance for the market penetration of EVs [Yu, Silva & Chu 2011]. Hybrid vehicles, vehicles which have both an electric motor and a gasoline engine, have been successful since they overcome the range anxiety of their owners by also running on gasoline. Since hybrids still require gasoline these vehicles do not fully mitigate the environmental consequences [Bradley & Frank, 2009].

Another refueling infrastructure design is to have quick battery exchange stations (BEs). These stations will remove a pallet of batteries that are nearly depleted from a vehicle and replace the battery pallet with one that has already been charged [Shemer, 2012]. This method of refueling has the advantage that it is reasonably quick. The unfortunate downside is that all of the vehicles serviced by the battery exchange station are required to use the identical pallets and batteries. It is assumed here that the developers of these battery pallets will coalesce around a single common standard, as has been the case for other car parts such as tires, wipers, etc. In conjunction to the

battery-exchange concept, it is assumed that there is a viable business model that provides a reasonable profit for companies that establish battery exchange facilities for the public. Battery exchange stations have been tried out by taxi vehicles in Tokyo in 2010 [Schultz 2010].

This paper discusses some logistical issues relevant to the design and operations of a fleet of EV vehicles operating within a battery-exchanging infrastructure from an operations research perspective. Besides the direct relevance to EV-based transportation, the models and analyses that may be developed in this research will intellectually contribute to the operations research, computer science and related fields. In addition, the models and methods being developed are also applicable to alternative fuels where empty tanks or canisters are exchanged for full ones, such as for hydrogen powered vehicles [Ogden, Steinbugler & Kreutz 1999].

## 2. Research Issues and Literature

Since EV technology is rapidly evolving, development of robust models for evaluating potential vehicle technologies and infrastructure designs could assist in decision making on the latter, and could make EVs more attractive and acceptable by car-owners, services and industries. Besides macro-economic, environmental, and governmental studies to argue the case for EV, there are many additional directions of research for understanding electric vehicles. Many of these are already being pursued by researchers around the world, including the following:

1. From a demographic perspective, extrapolation of the considerable research on acceptance and market penetration of hybrid vehicles [Zhou, Vyas, & Santini 2012, Axsen & Kurani 2009, Keith, Sterman, & Struben 2012],
2. Factors that drive preferences and market for EV [Woodjack et al 2012, Molin Stralen & Wee 2012, Chloud & Kagerbauer 2012]
3. Simulation based research on the impact of EV on traffic and energy [Knapen et al. 2012, Du Rakha, & Sangster 2012],

This paper focuses on logistics issues of EVs, including: routing vehicles, locating battery exchange stations, and determining their charging capacities. These problems are informally described in the next subsections. Some preliminary research ideas are discussed in Section 3.

### 2.1 Routing Issues

Taking a trip, especially one through lowly populated areas, requires the driver to plan when the vehicle will need to be refueled. Given the abundance of gasoline stations for standard vehicle, drivers usually consider refueling only when their fuel tank is low. The search for a good refueling point can be further aided by navigation systems and smart phone apps, such as Google Maps, that provide motorists the location of gasoline stations in the vicinity. In the case of electric vehicles, planning refueling is more important than for gasoline vehicles, since there are few places to recharge. Likewise, understanding routing would be even more critical for battery exchange facilities, since the infrastructure would gradually involve so, at least initially, the density of battery-exchange stations would be very low. Hence, one needs to develop models which look for the shortest routes from origins to destinations that include detouring when necessary. Objectives for these models could be to (a) minimize the total detouring distances and (b) minimize the total number of battery-exchange stops. The problem of finding the shortest path for an EV was originally discussed by Ichimori, Ishii and Nishida [1981], where a vehicle has a limited battery and is allowed to stop and recharge at certain locations. Lawler [2001] sketched a polynomial algorithm for its solution. If each arc requires an amount of fuel that does not depend on the length of the arc, and the goal is to find the shortest path constrained on the amount of fuel used (and the vehicle cannot stop to refuel), then the problem is exactly the shortest weight-constrained path problem [Garey & Johnson 1979].

### 2.2 Location issues

Suppose we wish to locate  $p$  battery-exchange stations in a place where there are currently none. The problem of optimally locating such refueling stations (battery recharging, battery exchanging and, other alternative refueling options can all addressed similarly) has been investigated by Kuby and collaborators [e.g., Kuby 2005, Kuby & Lim 2006, Upchurch Kuby & Lim 2009, Lim & Kuby 2010, Capar, Kuby & Rao, 2012]. Typically, they use modifications of flow capturing or flow interception models [Hodgson 1990, Berman Larson & Fouska 1992, Rebello, Agnetis & Mirchandani 1995], to cover as many Origin-Destination (O-D) routes as possible with a given number ( $p$ ) of stations. To compare proposed models, standard  $p$ -median and  $p$ -center problems [Mirchandani &

Francis 1990] have been used as proxies for maximizing proximity to stations and coverage by stations, respectively, for locating the stations. The developed models have been compared empirically for specific scenarios in order to choose one location model over another. However, these models do not take into consideration the likely possibility of vehicles making detours to refuel; therefore the direct consideration of locating facilities to minimize detouring distances and or minimizing detouring stops have not been included in the model developments.

Another location issue relates to the effect of locating battery exchange-stations on O-D traffic patterns of the driving population. If travelers from origins to destinations do not choose the shortest routes but instead chose routes that minimize their detouring costs due to refueling, then the addition of BE stations would change traffic patterns to a new equilibrium. There has been some recent consideration of the effect of EVs on traffic assignment and traffic equilibrium, but the research is only on restricting the distances EV can travel and assumes no refueling. [Jiang, Xie, & Waller 2012].

Another location issue is on the placement of BE stations for a fleet of electric service vehicles. An example is when a city or a regional bus company wishes to change their fleet to electric buses and needs to find the best locations for the BE stations. In this scenario, a bus would need to refuel after servicing a fewer number of trips than when the buses are gasoline powered. For a fleet of service EVs, battery exchange stations would be required instead of battery charging stations, since battery charging would require the buses to be idle for long periods of time which lowers the productivity of buses that are normally quite expensive. Battery-exchange stops will need to be planned in the bus itineraries since these battery exchange stops may require detouring (see the “scheduling issues” subsection). The locations of BE depots will significantly affect the productivity of transit operations, and therefore locating them optimally is critical when switching to electric buses.

### 2.3 Scheduling issues

The problems involved in scheduling a fleet of vehicles to service routes at specific times, for example a fleet of buses in a city, are broadly referred to as *vehicle scheduling problems* (VSP) which have been well studied and documented [e.g., Golden 1988, Laporte 2009, Freling, Wagelmans, & Paixão 1998]. Changing the fleet to one that uses electric vehicles increases the complexity of the problem because of the limited range of the electric charge on the batteries. This range limit will require the buses to refuel several times during their use throughout the day, sometimes having to detour significantly from the original route just to swap the batteries. Thus, despite the fact that EVs require less energy to operate than gasoline-powered ones, inefficiency is added by requiring frequent battery swaps. An explicit requirement of refueling, sometimes several times during a tour, adds not only the refueling requirements for the bus but also a detouring component to planning that is not present in the standard VSP problems. Electric buses which have battery pallets that can be swapped have been tested at the Shanghai EXPO [Zhu et al. 2012]. Clever scheduling of the EV fleet of buses to the routes and refueling stations can lower the amount of energy the buses use; this leads to an important new direction of research. This topic has been studied for fleets of delivery vehicles, where there is not an exact time requirement on the scheduling [Davis & Figliozzi, 2012].

### 2.4 Facility sizing issues

When a vehicle arrives at a BE station, it requests a fully charged battery pallet (an output of the station) to replace the nearly depleted batteries it currently holds. The request could either be satisfied by a fully charged battery pallet from the facility’s storage, or by a pallet that is just completing its charging. If the request is indeed satisfied, the vehicle in turn deposits a fully or partially spent pallet. If there are idle battery pallet chargers (BPC) at the station, the spent battery pallet is placed on a BPC and its recharging begins, otherwise it is kept in a queue until a BPC is available. If instead there is no fully charged battery available at the facility, then the vehicle could leave and go to a different facility (i.e., it balks). Alternatively, it could wait for a battery to fully charge, which may take some time. The bus could even take, if necessary, a replacement battery that is only partially charged and use that partially charged battery to travel to another battery-exchange facility on its route.

The size and attendant cost of the facility depends on both the number of BPCs it holds and number of battery pallets the facility keeps on hand. The availability of charged battery pallets at any given time depends on the size of the facility, the inventory of pallets, and demand for charged pallets the facility is experiencing. The facility incurs an indirect cost from the unavailability of charged pallets when an EV arrives for an exchange because the driver will not have to pay for a battery swap, and there may be a loss of goodwill from the unserved customer. Models to evaluate total direct and indirect costs for possible decisions on facility sizing and inventory holding would be very important in designing the BE infrastructure.

### 3. Some Preliminary Research

#### 3.1 EV Routing Problems

We will assume a network model. The prototypical problem is to find the “best” route from a given origin to a given destination, possibly making battery pallet exchanges (we will refer this generically to “refueling” when the context is clear), so that no segment without refueling is greater than a given range  $c$ . Initially consider the objective by which we choose the “best” to be to *minimize travel distance*.

Let  $G = (V, E)$  be a network with node set  $V$  and arc set  $E$ . Let vertices  $s, t \in V$  represent the starting and ending points of a trip by the EV. Let  $d_{ij}$  denote the length each arc  $(i, j) \in E$  and let  $R \subseteq V$  be the given set of BE locations. Let  $p$  be the maximum number of times we are allowed to refill on the trip. We define the *EV shortest walk problem* (EV-SWP) as the problem of finding the shortest route in  $G$  starting at  $s$  and ending at  $t$  such that any walk contained in the route starting and ending at nodes in  $\{s, t\} \cup R$  has length at most  $c$ . This route may contain cycles since detouring for a battery exchange may result in a cyclic detour. Since we are constrained by the number of stops, this route may contain at most  $p$  elements of  $R$ , since a vehicle would never visit the same BE twice. This problem can be seen as a modification of the classic *constrained shortest path problem* [Garey & Johnson 1979]. The EV-SWP can be formulated and solved as a linear integer program using standard off-the-shelf optimization packages [Adler & Mirchandani 2012]; however, such a formulation requires non-polynomial solution time.

If there is no limit on the number of stops (i.e. the vehicle can make up to  $|R|$  stops) and the only concern is to minimize total distance, then it can be shown that the problem can be easily solved in polynomial time using the standard shortest path labeling algorithm [Ahuja, Magnanti & Orlin. 1993], albeit up to  $|R|$  times. We will first describe the algorithm with an illustration before we analyze it. Suppose we wish to travel from vertex  $s$  to vertex  $t$  in the network of Figure 1. The color nodes in the figure are refueling stations.

When the range  $c$  is large, say greater than 50, then the shortest path from  $s$  to  $t$  can be found using a shortest path algorithm such as Dijkstra’s [Ahuja, Magnanti & Orlin. 1993]; the bold path shown in Figure 1 is the shortest path of length 39. Note that this path does not pass any refueling points.

If the range was 20 then the vehicle will have to refill at least once to reach vertex  $t$ . The shortest path tree from  $s$  to all reachable nodes within distance  $c$  is shown in Figure 2. As shown in the figure, two stations are reachable from  $s$ , the green station at vertex 5 and the mauve station at vertex 9.

We then can do the same with these two stations as the starting points, and continue this process hoping from station to station. This results in a network of range-limited shortest paths between stations, origin and destination, where each of the arcs correspond to an path in a range-limited shortest path tree. We refer to this undirected network as the *refueling shortest path network* (RSPN) denoted by  $G' = (V', E')$ , and let  $|V'| = n'$  and  $|E'| = m'$ . Observe that RSPN can be obtained in, at most,  $(|R| + 1)$  iterations of the shortest path algorithm: one iteration for the starting node and one for each of the stations. Figure 3 shows the RSPN for the example.

Now the shortest path in this network is  $s - 5$  (green station) – 13 (orange station) –  $t$  with a distance of 43. This corresponds to EV walk in the original network  $s-1-4-5(\text{refuel})-4-7-13(\text{refuel})-14-t$ . The paths  $s - 5$ ,  $5 - 13$ , and  $13 - t$  are each subtrips in the solution. Note that the walk includes a cycle 4-5-4 and a detour 7-13-14 as compared to the shortest segment 7-12-14 in the shortest path from  $s$  to  $t$  (see Figure 4).

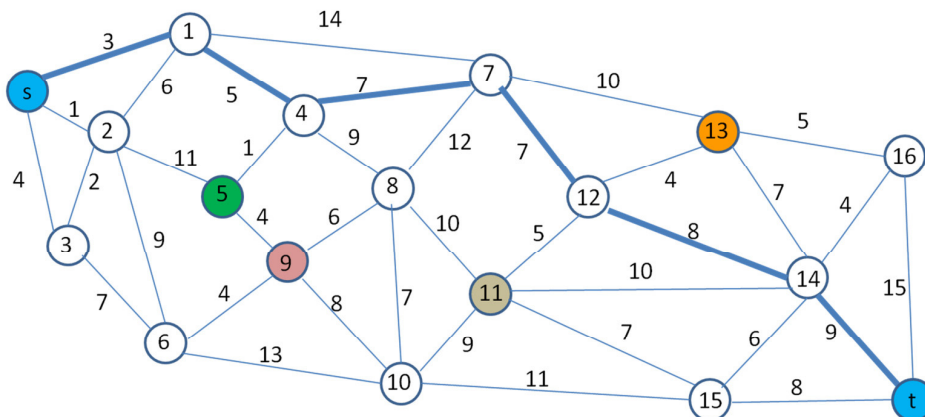
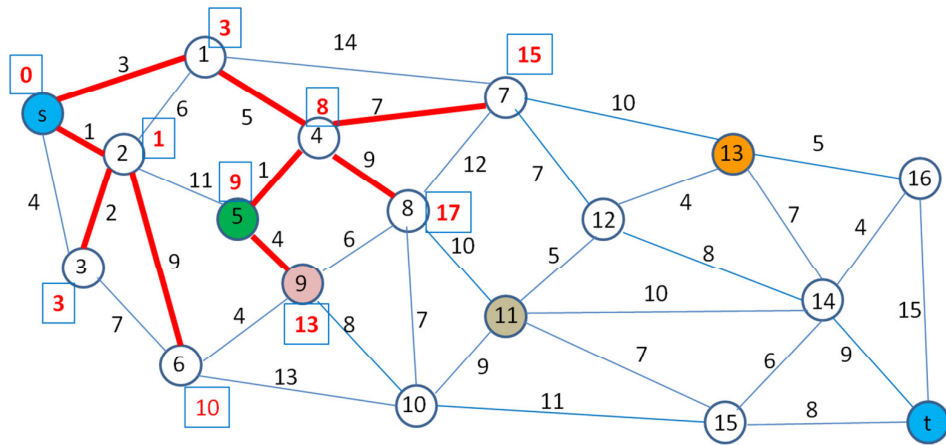
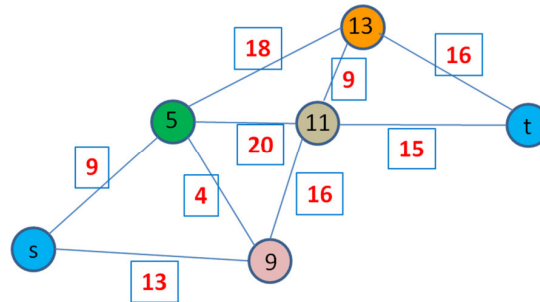


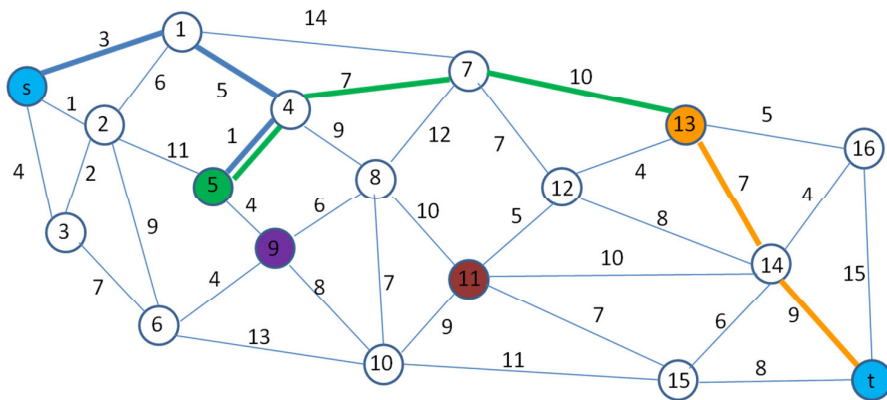
Figure 1: Shortest unconstrained path from origin  $s$  to destination  $t$



**Figure 2** The bold lines indicate the range-limited shortest path tree from  $s$ .  
The boxed labels are the distances to reachable nodes from  $s$ .



**Figure 3** Refueling Shortest Path Network, *RSPN*, for the example.  
The boxed numbers are the lengths of the shortest feasible paths.



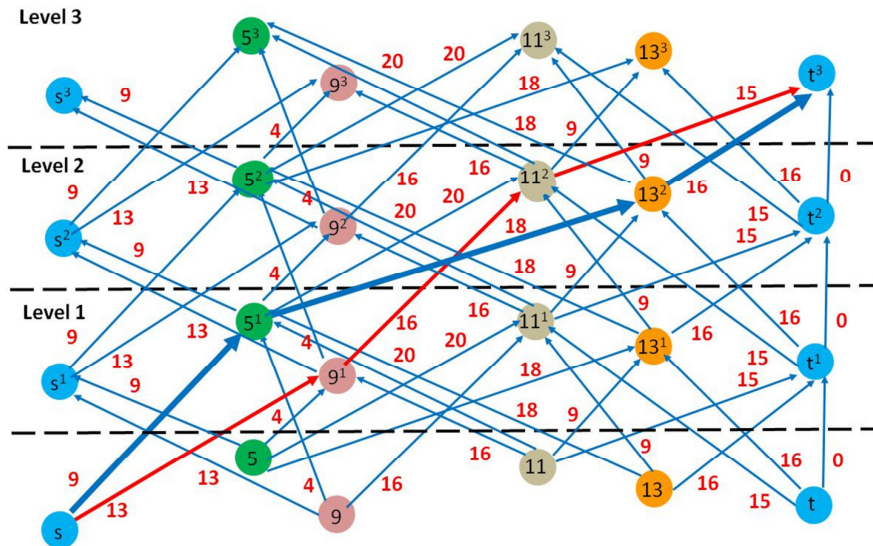
**Figure 4** EV shortest walk for the example

We can prove the following theorem [Adler & Mirchandani 2012]:



**Theorem 1** The EV-shortest walk problem can be solved in  $\mathcal{O}(|R| (n \log_2 n + m))$  time.

**The restricted shortest EV-walk problem:** The more interesting cases of EV-SWP are when refueling stops are penalized with a “stop cost” representing something such as the price of each battery exchange. Then we need to consider both the distance cost and the stop cost to find the optimal route. When stops are very costly then the



**Figure 5** Multilevel network when the number of refueling stops is restricted to a maximum of two

problem becomes that of minimizing the number of battery exchange stops to reach the destination. So now we will create a directed graph where the vertices represent the refueling stations as well as  $s$  and  $t$ , and directed edges represent paths from stations and the start/end nodes to other stations and start/end nodes that are reachable with fuel  $c$ . If we are restricted to  $p$  refueling stops, then  $p + 2$  copies of these directed arcs are needed. One may think of each network copy signifies the reachable nodes with a fully-charged battery (or with a full fuel tank). The RSPN for the example gives the multi-level network shown in Figure 5 when we are restricted to a maximum of 2 refueling stops.

We create a directed graph  $G'' = (V'', A'')$  that is a transformation of RSPN,  $G' = (V', A')$ . The vertex set  $V'' = \{x^{[i]}: x \in \{s, t\} \cup R, i \in \{0, 1, \dots, p + 1\}\}$  has  $p + 2$  copies of each vertex in  $\{s, t\} \cup R$ . We define the arc set as  $A'' = A_1'' \cup A_2''$  where  $A_1'' = \{(a^{[i]}, b^{[i+1]}): (a', b') \in E', i \in \{0, 1, \dots, p\}\}$  and  $A_2'' = \{(t^{[i]}, t^{[i+1]}): i \in \{0, 1, \dots, p\}\}$ . The distance mapping is defined for arcs in  $A_1''$  as  $d''(a^{[i]}, b^{[i+1]}) = d'(a', b')$  and for arcs in  $A_2''$  as  $d''(t^{[i]}, t^{[i+1]}) = 0$ . Thus, the distances in this new graph between levels are the same as those between vertices in the RSPN, except for the addition of zero distance edges which allow the vehicle to go from any of the  $t^{[i]}$  nodes to the  $t^{[p+1]}$  node penalty free. This graph has the property that any path from  $s^{[0]}$  to  $t^{[p+1]}$  contains exactly  $p + 1$  arcs, and corresponds to a path in  $G'$  that travels from  $s$  to  $t$  in at most  $p + 1$  edges. Thus, to find the stop-limited shortest walk, we need to find the shortest path in  $G''$  from  $s^{[0]}$  to  $t^{[p+1]}$ . The shortest path will contain exactly  $p$  intermediate nodes, and the number of refueling stations the vehicle will stop at corresponds to the number of intermediate nodes in the path until reaching the first termination node  $t^{[i]}$ . The bold edges in Figure 6 show this shortest path for the example:  $s^0$  to  $5^1$  (stop at green station),  $5^1$  to  $13^2$  (stop at orange station) and then to destination  $t^3$ , with the travel distance 43 as discovered earlier. For illustrative purpose we have indicated another path from  $s^0$  to  $t^3$ :  $s^0$  to  $9^1$  (stop at mauve station),  $9^1$  to  $11^2$  (stop at brown station) and then to destination  $t^3$ , with the travel distance 44 which does not correspond to a shortest walk.

We can prove that even this problem can be solved in polynomial time [Adler & Mirchandani 2012]

**Theorem 2:** The  $p$ -stops limited EV-shortest walk problem can be solved in  $\mathcal{O}(p|R| (n \log_2 n + m))$  time.

### 3.2 EV Fleet Scheduling Problems

In the context of EV public transportation, there are several important and interesting optimization problems when we change the fleet from gasoline-powered buses to EV buses. Given a set of bus routes that need to be serviced, and a set of BE stations for the new EV buses, we need to be able to find the best way to assign buses to routes. The limited range of each charged battery pallet, as well as the small number of BE stations, means that it is important to plan exactly where and when the buses will have their batteries exchanged.

The classic *Single Depot Vehicle Scheduling Problem* (VSP), in the context of public transit, may be defined as follows: given a depot at location  $d$  and  $n$  trips to be serviced, with  $j^{\text{th}}$  trip starting at location  $s_j$  and ending at locations  $e_j$  with corresponding starting and ending times  $st_j$  and  $et_j$  for  $j = 1, \dots, n$ , and given traveling times and costs for all pairs  $(e_i, s_j)$ ,  $(e_j, d)$ , and  $(d, s_j)$ , find the minimum cost assignment of buses to trips, such that every trip is served by exactly one bus. Here each trip could be considered as a job with a given start and end time and a start node and end node on a graph. Later we will represent each trip as a single node on a graph, and arcs between two nodes will represent if two trips can be serviced by the same bus. This problem has several polynomial time approaches to solve it [e.g., Golden 1988, Laporte 2009]. However, upon adding the constraint that each bus can only travel a certain distance before needing to visit a BE station the problem becomes NP-hard. The addition of the limited range constraint changes the problem into what we shall refer to as *Electric Vehicle Scheduling Problem* (EV-VSP), which is what we plan to address.

Formally the single depot EV-VSP is the follows. We are given a depot  $d$  at location  $ds$ ,  $k$  BE stations at locations  $FS = \{fs_1, fs_2, \dots, fs_k\}$ , with EV buses needing to service  $n$  trips, where  $j^{\text{th}}$  trip is from location  $s_j$  to location  $e_j$  for  $j \in N$ ,  $N = \{1, \dots, n\}$ , having for start and end times  $st_j$  and  $et_j$ , and electric charge requirements  $f_j$ . Each bus has an electric charge capacity  $w$  that gives it a limited range. For each dead-heading trip  $\{(a, b): a, b \in FS \cup d \cup \{s_j: j = 1, \dots, n\} \cup \{e_j: j = 1, \dots, n\}, a \neq b\}$ , we have a travel time  $t_{ab}$ , cost  $m_{ab}$ , and energy consumption  $f_{ab}$ . The EV-VSP problem is to find a feasible minimum cost assignment of buses to trips, and the BE stations between trips being serviced, such that each trip is serviced by exactly one bus, each bus route starts and ends at the depot, and any route a bus takes between two refuel stops, or the depot and a refuel stop, it uses at most  $w$  charge. For meaningful application, but with no loss of generality, we assume that at most one BE station will be visited between two trips, a BE station may be visited between the depot and the first trip and/or after the last trip before returning to the depot. Each bus starts at the depot with a full charge; if a depot also has charging facilities a BE station can be placed in the same location as the depot, although this is not assumed by default. The EV-VSP is a generalization of VSP since the single depot vehicle scheduling problem is a special case of where  $w = \infty$  and  $k = 0$ . The EV-VSP has many similarities to the capacitated arc routing problem; Wøhlk [2008] gives an overview.

Without loss of generality, assume that the trips are ordered by their start times (so  $st_i \leq st_j$  for  $1 \leq i \leq j \leq n$ ). We can create a directed graph  $G = (V, A)$ , cost and fuel requirement functions  $c, f: A \rightarrow \mathbb{R}^+$ , and a constant value  $w$  to represent the problem. This representation of the problem fully captures the time compatibility component in the connections of  $G$ . We say that trips  $i, j$  are *compatible* if  $et_i + t_{ij} \leq st_j$ . Two trips  $i, j$  are compatible with BE station  $l$  if  $et_i + t_{il} + t_{lj} \leq st_j$ . We write the relationship for compatibility of  $i$  and  $j$  as  $comp(i, j)$  and  $i$  and  $j$  are compatible with BE station  $l$  as  $comp_f(i, j, l)$ . Define the set  $H$  as:

$$H = \left\{ h_{ab}^l: a, b \in \{1, \dots, n\}, a < b, l \in \{1, \dots, k\} \right\} \cup \left\{ h_{ad}^l: a \in \{1, \dots, n\}, l \in \{1, \dots, k\} \right\} \\ \cup \left\{ h_{da}^l: a \in \{1, \dots, n\}, l \in \{1, \dots, k\} \right\}$$

Set  $H$  represents the possible BE station visits that could occur between trips. So  $h_{ab}^l \in H$  represents a bus stopping to exchange batteries at BE station  $l$  after servicing trip  $a$  but before servicing trip  $b$ . We now define node set  $V = N \cup H \cup \{d\}$ . The arcs of  $G$  as well as definitions of  $c$  and  $f$  are given in Table 1:



Set	Definition	Fuel	Cost
$A_1$	$\{(a, b) : a, b \in N, \text{comp}(a, b)\}$	$f_a + f_{ab}$	$c_{ab}$
$A_2$	$\{(a, d) : a \in N\}$	$f_a + f_{ad}$	$c_{ad}$
$A_3$	$\{(a, d) : a \in H\}$	$f_{ad}$	$c_{ad}$
$A_4$	$\{(d, a) : a \in N \cup H\}$	$f_{da}$	$c_{da}$
$A_5$	$\{(a, h_{ab}^l) : a \in N, b \in N, l \in H, \text{comp}_f(a, b, l)\}$	$f_a + f_{al}$	$c_{al}$
$A_6$	$\{(h_{ab}^l, b) : a \in N, b \in N, l \in H, \text{comp}_f(a, b, l)\}$	$f_{lb}$	$c_{lb}$
$A_7$	$\{(a, h_{ad}^l) : a \in N, l \in H\}$	$f_a + f_{al}$	$c_{al}$
$A_8$	$\{(h_{ad}^l, d) : a \in N, l \in H\}$	$f_{ld}$	$c_{ld}$
$A_9$	$\{(d, h_{da}^l) : a \in N, l \in H\}$	$f_{dl}$	$c_{dl}$
$A_{10}$	$\{(h_{da}^l, a) : a \in N, l \in H\}$	$f_{la}$	$c_{la}$

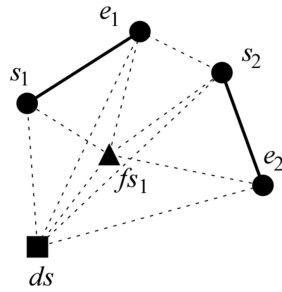
Table 1: The arcs in graph  $G = (V, A)$  where  $A = \bigcup_{i=1}^{10} A_i$ 

Figure 6: An instance of the EV-VSP

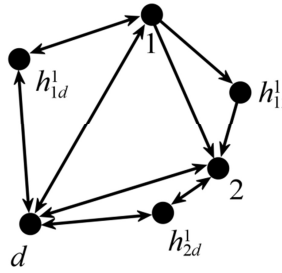


Figure 7: The corresponding graph G

Arc set  $A_1$  represents a bus taking a dead-heading trip between two trips. The fuel requirement for the origin trip is added to the fuel cost of the dead-heading trip so that we do not need to associate fuel requirements with vertices. Arc sets  $A_2$ ,  $A_3$ , and  $A_4$  represent dead-heading trips between the depot and trips or refuel stations. Arc sets  $A_5$  and  $A_6$  represent dead-heading trips between trips and fuel stations. Arc sets  $A_7$  and  $A_8$  represent traveling from a trip, refueling, then returning to the depot. Arc sets  $A_9$  and  $A_{10}$  represent traveling from the depot to a refueling station then from a refueling station to a trip. An example G is given in Figures 6 and 7.

Given  $G$ ,  $c$ ,  $f$ , and  $w$ , The EV-VSP is now the following: find a minimum cost set of cycles  $C$  in  $G$  that visit each node in  $N \subset V$  exactly once, where each cycle includes vertex  $d$  and no induced walks of  $C$  between starting and ending at vertices in the set  $\{d\} \cup H$  and containing no intermediate vertices of  $\{d\} \cup H$  have a fuel requirement greater than  $w$ . Not only can EV-VSP be shown to be NP-hard, but the authors [Adler 2013] have recently shown that even finding a solution that is guaranteed to be within 150% of optimal is NP-hard. They have also developed a branch-and bound approach and an heuristic to solve EV-VSP. Preliminary computational experiments show, as expected, the stopping for battery exchanges require more travel, more EV buses, and fewer service trips per buses when comparing with traditional gasoline-powered buses.

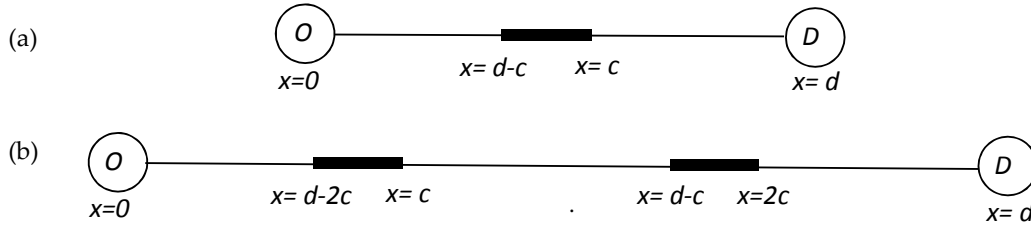
### 3.3 Location Problems for BE Facilities

We can define three prototypical location problems that need consideration of refueling detours: (1) Location of BE exchange stations for minimizing the total detouring for given origin-destination demands, (2) location of BE stations to obtain a better traffic equilibrium from the perspective of energy usage and air quality, and (3) location of BE stations for a fleet of service electric vehicles, for example, a fleet of EV buses for public transit. We will assume underlying shortest routes (EV-SWP) between network points as described above.

Consider the first problem of locating BE facilities for a given set of O-D demand, and the case when the transportation network is a tree network. The location models and algorithms developed for trees can then be extended to general networks. We begin by considering the simple special case where we have an EV planning to travel between two points  $O$  and  $D$  along a road. Let  $c$  be the maximum distance the vehicle can travel before

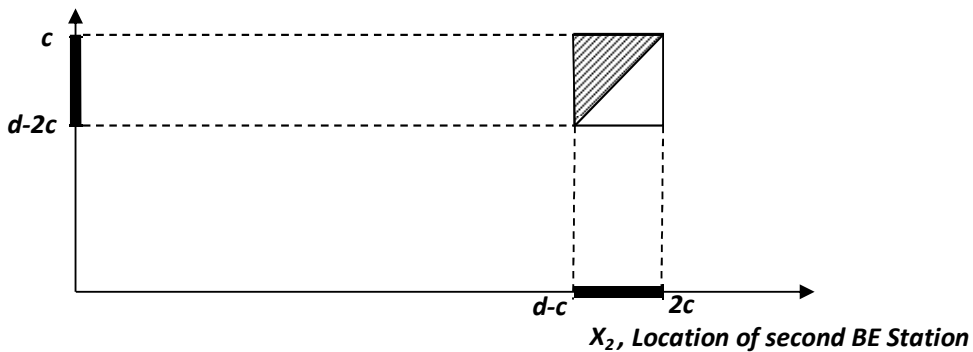
needing to refuel and let  $d$  be the length of the road, where  $d > c$ . We are interested in finding the optimal locations for BE stations so that the electric vehicle can travel between the two points. We can describe points along the route by their distance from starting point  $O$ .

If  $c < d < 2c$ , then we only need a single BE station. That refueling station can fall anywhere along the interval  $[d - c, c]$  (see Fig 8-a). If  $2c < d < 3c$  we need to place two BE stations along the route, where the first one falls in the interval  $[d - 2c, c]$ , and the second in the interval  $[d - c, 2c]$  (see Fig 8-b).



**Figure 8:** Illustration of localization sets for BE facilities on a single route

**$x_1$ , Location of first BE Station**



**Figure 9:** Localization polyhedral for two facility case.

The optimal refueling station placements are not independent of each other since the distance between the two stations must be at most  $c$ . If you consider the refueling station locations as a point in space  $[0, D]^2$ , the feasible solutions form a convex polyhedral (see Fig 9). Depending on what we try to optimize, the optimal solution may fall at an extreme point of the convex polyhedral or in an interior point. For example, if the EV driver prefers to maximize the lowest charge level the EV vehicle will have with only two stops, then interior point  $(\frac{d}{3}, \frac{2d}{3})$  will be optimal. If the driver is indifferent to level of charge as long as she reaches  $D$ , then all points in the polyhedral are equally optimal; in particular the extreme points are optimal, which are computationally easy to identify.

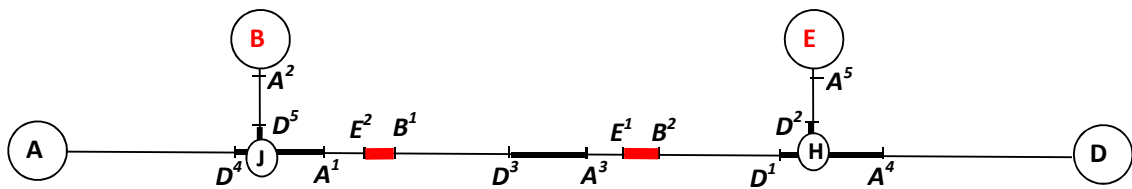
Generalizing to a path of any length, suppose the minimum number of BE facilities required is  $p$ , using the above arguments. Then the feasible space can be shown to be a polyhedral in  $p$ -space. Define location variables as  $x_1, \dots, x_p$  for some  $p \in \mathbb{Z}^+$ , where  $x_i \in [0, D]$  for  $i = 1, \dots, p$ . Then the location model can be formalized as follows

$$\begin{aligned} & \text{minimize } Z(x_1, x_2, \dots, x_p) \\ & \text{s. t. } x_i + x_{i+1} \leq c \quad \forall i = 1, \dots, p-1 \\ & \quad x_i < x_{i+1} \quad \forall i = 1, \dots, p-1 \end{aligned}$$

where  $Z(x_1, x_2, \dots, x_p)$  is the objective function representing the cost function of the driver. If  $Z$  is a linear or concave function, then a solution will lie at an extreme point of the polyhedral. If  $Z$  is convex and differentiable, then the solution may lie on an interior point.

Now consider the situation where we have  $n$  points  $v_1, \dots, v_n$ , and a collection of OD flows connecting these points. This will create a tree network where the leaves of the tree are  $v_1, \dots, v_n$ . A problem of interest is to find the minimum number of refueling stations and their locations such that the total detouring cost for the OD flows is minimized. Each vehicle may travel between any pair  $(v_i, v_j)$  for  $i, j \in \{1, \dots, n\}$ , for some cost function  $Z$ . We may then narrow down the search space with appropriately defined segments between break points. (See example in Figure 10). The search for optimal placement of the EV stations, along with the study of the complexity of the problem and the possible solution algorithms, would be both intellectual and relevant research issues. This could then lead to the more general location problem on general networks, either from the modeling sights gained from the tree case or the algorithmic methods from solving for tree case.

The second location problem is to optimally locate BE facilities on a network so that the resulting equilibrium flow patterns are “good” from the perspective of energy usage and air quality. An approach to address this problem is to consider a bi-level problem architecture, where at the higher level we consider the location decisions, and the lower we consider the problem of finding the traffic equilibrium flows. The upper level problem solution would perhaps be based on the location model discussed above, while the lower level solution could be based on iteratively



**Figure 10:** Localization segments for a tree network

using one of several methods available in the literature [e.g., Florian 1976, Sheffi 1985, Larsson & Parkinson 1999, Nie, Zhang, & Lee 2004].

The third location model identified above deals with locating BE stations for a fleet of EVs. Assume we are given a set of bus trips and we are interested in serving them with EV buses, like in the EV-VSP defined in section 3.2. Now however, we add variables representing BE station locations. Instead of  $FS = \{f_{s_1}, f_{s_2}, \dots, f_{s_k}\}$  being fixed, they are variables that we would like to adjust to minimize the overall cost to the system. We are given a cost function  $C_f: FS \rightarrow \mathbb{R}$  which assigns a cost to placing a BE station at a location. The travel times and fuel costs between trip start and end locations and refuel stations are no longer fixed; they are also functions of the BE locations. A simplifying assumption would be to have these functions depend only on the distance between the dead-heading trips, however depending on the real-world considerations that may not be realistic. We will refer to this problem as the *Battery Exchange Station Location Problem for Transit* (BESLPT). Since the EV-VSP is NP-hard, it follows that BESLPT is also NP-hard, because it contains EV-VSP as a subproblem. Still, we can study special cases, and develop exact and/or approximate algorithms for special and general cases respectively.

### 3.4 BE Station Sizing Problem

At first glance, it would appear that one may be able to model arrivals of EVs with spent charge at a facility as a non-homogeneous Poisson process and the service times for exchanging the batteries as a deterministic. Hence the charging system may be approximately treated as an M/D/p queuing system, for which there are several analytical results for periods when the arrival rate is homogenous [Tijms 1994, Franx 2001].

If the request for a recharged battery pallet is satisfied, the vehicle, in turn, deposits a fully- or a partially-spent pallet. If there is an idle battery pallet chargers (BPC), the spent pallet is assigned to it. Otherwise, it is kept in a queue for an available BPC.

If there is no fully charged battery available, then the vehicle could leave and go elsewhere (i.e., it balks), or it could wait for a fully charged battery and experience some discomfort, or it could pick up a partially charged pallet with enough charge to reach the next BE station on its planned route. Thus the existence of two interacting queues,

EVs waiting for battery pallet swaps and spent pallets waiting for a BPC. To the authors' knowledge, analytical models for such a system are not available.

The authors have built a preliminary simulation model, with some additional realistic considerations such as that the driver will wait for a few minutes if a charged battery will soon be available. The simulation showed, as expected, fewer customers leave before being served if (1) number of BPCs are increased, (2) number of battery-pallets in inventory are increased, and (3) if demand for pallets decreases. Development of analytical models that provide similar results could then be used in an optimization framework that both locates and sizes BE facilities to minimize overall costs that includes fixed costs of facilities, costs to drivers for detouring, service-related costs (e.g., waiting time for a battery exchange), and energy and environmental society costs.

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